Hypothesis for the Origin of Cross-Hatching

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On the basis of experimental evidence, a flow model is formulated assuming the presence in the boundary layer of a stationary array of regularly spaced longitudinal vortices. It is demonstrated that this assumption alone implies the presence of spatially periodic pressure fluctuations (having maxima along Mach lines) capable of producing the crosshatch patterns. The vortices are assumed to originate from surface irregularities near the leading edge; the probability of their appearance is enhanced by the presence of a local region of concave curvature in the boundary-layer streamlines. According to the hypothesis, surface ablation is not a necessary condition for the presence of the pressure fluctuations that lead to cross-hatching, but may be the mechanism causing the streamline curvature. For cross-hatching to appear, the boundary layer aft of the concavity must be turbulent and the local flow supersonic.

Introduction

THE discovery that the surfaces of several recovered entry vehicles were engraved with very regular patterns of crossed grooves (now commonly called cross-hatching) has inspired a determined effort to understand their meaning. Canning and his co-workers[1–3] drew attention to the occurrence of similar markings on models recovered from free-flight ranges. Larson and Mateer[4,5] demonstrated that cross-hatching could be reproduced on ablating models in a high speed wind tunnel when surface pressures were made sufficiently high. Following this demonstration, Larson and Mateer[5] and Laganelli and Nestler[6] carried out detailed experimental studies that enabled the drawing of a fairly complete physical picture of the phenomenon. More recent results are available in Ref. 7. It is now reasonably certain that cross-hatching appears only when the local flow is supersonic and when the boundary layer is turbulent. The phenomenon occurs on wedges as well as on cones and, with varying scale, for a variety of ablative surface materials. Whether ablation itself is a necessary condition, however, or merely a means of recording the event is not yet known.

The intriguing regularity of the patterns has provoked researchers to suggest a variety of possible mechanisms for their origin. In Ref. 3, evidence is cited suggesting the presence of arrays of longitudinal vortices, and it is conjectured that the presence of vortices may be connected with the origin of cross-hatching. In Ref. 5, it is conjectured that a system of standing shock waves, resulting from a coupling between gas dynamics and ablation, may be responsible for the occurrence of cross-hatching. These conjectures are presented without any supporting analysis, however.

In this paper, we propose a theoretical model for the origin of cross-hatching which, while incomplete, promises to reconcile these conjectures and to lend itself to further test. As in Ref. 3, we take seriously the evidence pointing to an array of stationary longitudinal vortices. Assuming their presence, we show that this assumption alone implies the presence of standing waves capable of producing the cross-hatch patterns. We then undertake to study why an array of longitudinal vortices may be present and capable of sustaining itself. By adapting Maskell's concept of limiting streamlines,[8] we attempt to show how irregularities in the surface at the leading edge can lead to the formation of an array of longitudinal vortices.

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Analysis

Flow Model

Consider supersonic flow over a two-dimensional wedge. Assume that, through some mechanism to be discussed later, there exists within the boundary layer a stationary array of counterrotating longitudinal vortices regularly spaced in the spanwise direction. Introduce a displacement surface,[9] defined as being that surface which, in a completely inviscid flowfield, produces the same external flow as that actually existent outside the boundary layer. Whatever the location of the displacement surface, it is clear that because of the presence in the boundary layer of the array of vortices, the normal velocities at the displacement surface must show a spatially periodic wave form in the spanwise direction. Accordingly, we are led to postulate that the shape of the displacement surface running spanwise likewise must be periodic to match the normal velocities. The period of the wave form must of course be commensurate with the spacing of the vortices. The postulated shape of the displacement surface is illustrated in Fig. 1. Above the displacement surface the flow by definition is inviscid. Therefore, we may estimate the pressure on the displacement surface by means of linearized wing theory. Assigning coordinates as illustrated in Fig. 1 (where x is directed along the wedge surface), we obtain the pressure coefficient from[10]

$$C_p = \frac{p - p_\infty}{q_\infty} = \frac{2}{\pi}\frac{\partial}{\partial x}\iint_\tau \frac{s(x_1,z_1)dx_1dz_1}{[(x - x_1)^2 - \beta^2(z - z_1)^2]^{1/2}} \tag{1}$$

Here, $s(x_1,z_1)$ is the streamwise slope of the displacement surface and τ is the area enclosed by the Mach forecone from the field point x,z. Assuming a turbulent boundary layer, for which the displacement thickness δ^* grows approximately linearly with increasing x, we represent the postulated displacement surface by

$$\delta^*(x,z) \approx x(\lambda_m + \lambda_0 \cos\alpha z) \tag{2}$$

The streamwise slope of the displacement surface is then

$$s(x,z) = \partial\delta^*/\partial x = \lambda_m + \lambda_0 \cos\alpha z \tag{3}$$

Inserting Eq. (3) in (1) and integrating yields

$$C_p = \frac{2\lambda_m}{\beta} + \frac{2\lambda_0}{\beta}\cos\alpha z J_0\left(\frac{\alpha x}{\beta}\right);\ \beta = (M_\infty{}^2 - 1)^{1/2} \tag{4}$$

The pressure on the displacement surface is of course transmitted to the wedge surface. Equation (4) approximates the additional pressure induced on the wedge surface by the presence of the boundary layer. It is important to note that although the streamwise slope of the displacement surface has been assumed to vary periodically only in the spanwise direction, nevertheless the induced pressure varies periodically both spanwise and longitudinally. To see this more clearly, let us replace the Bessel function J_0 by its asymptotic form. The periodic part of the induced pressure becomes

$$\Delta C_p = \frac{2\lambda_0}{\beta}\left(\frac{2\beta}{\pi\alpha}\right)^{1/2} x^{-1/2}\cos\alpha z \cos\left(\frac{\alpha x}{\beta} - \frac{\pi}{4}\right) \quad (5)$$

Letting $x = \xi + \beta\pi/4\alpha$, we see that relative maxima in ΔC_p will occur where $\cos\alpha z \cos\alpha\xi/\beta = 1$, or where

$$\left.\begin{aligned}\alpha z &= 2m\pi\\ \alpha\xi/\beta &= 2n\pi\end{aligned}\right\} m,n = 0, \mp1, \mp2,\ldots \quad (6)$$

and where

$$\left.\begin{aligned}\alpha z &= (2m+1)\pi\\ \alpha\xi/\beta &= (2n+1)\pi\end{aligned}\right\} m,n = 0, \mp1, \mp2,\ldots \quad (7)$$

The relative maxima occur at spaced points connected by Mach lines as shown on Fig. 2. Note that along the Mach lines connecting the relative maxima, ΔC_p is never negative. The induced pressures are greatest for relatively small values of x, diminishing as $x^{-1/2}$. When ablation begins, the surface at points of maximum pressure near the leading edge will recede the most rapidly. As the surface recedes at these points, leaving pockmarks, Mach waves propagating from the pockmarks will move downstream and be reinforced each time they intersect downstream at a point of maximum pressure. In this way, the lines joining the pressure maxima will preserve themselves on the surface, causing the appearance of cross-hatching. The crossed lines will first appear at small x and work their way rearward as ablation proceeds. This behavior is in accord with the experimental observations.

It is possible to envision how the process just described might be calculated. Figure 3 is a section of the flow taken along a line that runs through a set of pressure maxima in the streamwise direction (e.g., line AB in Fig. 2). As is convincingly demonstrated in Ref. 9, the pressures on the displacement surface are transmitted through the boundary layer to the wedge surface not along normals to the wedge surface but along continuations of the isobars (i.e., Mach lines) in the inviscid external flow. The transmission of the pressure maxima in this way is illustrated in Fig. 3. Now let the body begin to ablate, and assume that the flow adjusts so rapidly to the changing surface that at any given time the flow may be considered to be the steady-state flow corresponding to the boundary conditions at that time. Consider a time shortly after the beginning of ablation. The surface at points a,b,c, where the original induced pressures are maxima, will have receded more than at neighboring points. Hence, the surface will have become wavy, as shown by the dotted line in Fig. 3. Since the surface has now changed shape, a new displacement surface must be determined. It is clear that the new surface will have not only a spanwise periodic deformation but also a streamwise periodic deformation. This is shown in the figure. The slopes of the new displacement surface will be greatest in the vicinity of the original pressure maxima. Hence, the new induced pressures, transmitted along essentially the same isobars, will again have maxima at the same points on the surface as the original induced pressures. These maxima will exceed the original maxima in magnitude, reflecting the increased slope of the displacement surface. Carried out step by step, the calculation would show that the crosshatch pattern engraved on the surface by the original induced pressures would be reinforced at each step, at least during the initial stages of ablation.

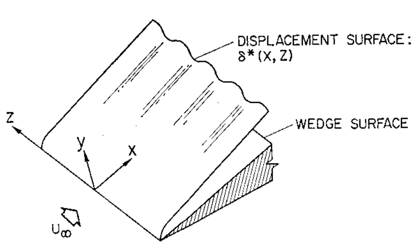

Fig. 1 Postulated displacement surface.

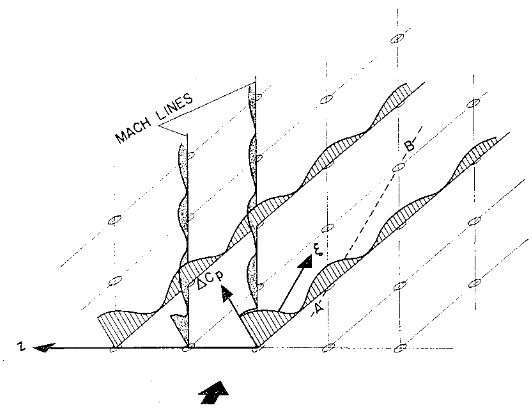

Fig. 2 Pressure maxima.

According to the hypothesis, ablation is not a necessary condition for the presence of the pressure variations that lead to cross-hatching, but is a means of recording, reinforcing, and spreading the pattern once it appears. The hypothesis is in accord with two of the three conditions necessary for the appearance of cross-hatching postulated in Ref. 5 on the basis of the observations. Obviously, a supersonic flow is necessary. A turbulent boundary layer would appear to be a necessity in order that pressure changes across the Mach waves remain sharp near the surface. As for the third condition, that of a high surface pressure, the hypothesis would suggest that a high surface pressure may be a necessity for the recording of the pattern, not for its presence. The capacity of the surface to respond (by differential recession) to even small pressure differences is enhanced as the surface pressure is increased.

Evidence of Longitudinal Vortices

It has been shown that the appearance of cross-hatching can be explained by a relatively simple argument if the presence in the boundary layer of a regular array of counterrotating longitudinal vortices is admitted. The burden of the argument therefore rests on evidence pointing to the presence of such an array of vortices.

There is abundant physical evidence that arrays of longitudinal vortices were in fact present not only in the experiments where cross-hatching was observed, but in numerous other experiments involving boundary-layer flows over plane surfaces. This is to suggest that the appearance of longitudinal vortices in certain types of boundary-layer flows may be a common occurrence. For example, Ginoux[11] reports

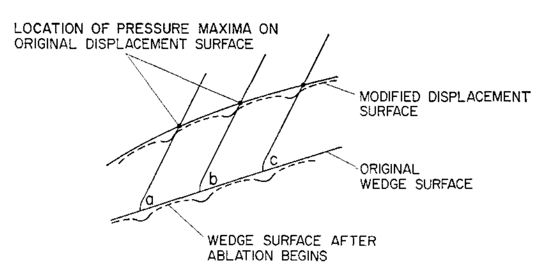

Fig. 3 Deformation of displacement surface.

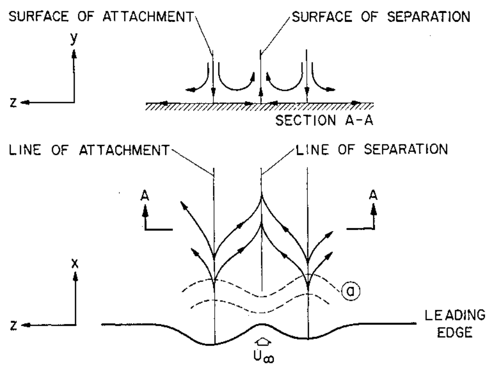

Fig. 4 Lines of attachment and separation.

having found periodic spanwise fluctuations in Pitot-pressure measurements in the boundary layer over plates with backward-facing steps. In Ref. 12, fluorescent oil photographs were presented to . . . "show evidence of streamwise vortices being produced at a Mach number of 3.03 on a flat plate with no steps or protuberances, behind a rearward-facing step, behind a forward-facing step, and behind a wire roughness." A large number of additional references could be cited (c.f., in particular, Ref. 13). As for the experiments where cross-hatching was observed, many (though not all) of the models tested show clear evidence, in the form of longitudinal grooves, of the presence of regularly spaced longitudinal vortices. In the first experiment of Larson and Mateer,[4] demonstrating that cross-hatching could be produced on wind-tunnel models, the camphor cone used in the demonstration had regularly spaced longitudinal grooves engraved on its surface both at a low surface pressure where no cross-hatching was observed, and at a higher surface pressure where cross-hatching was observed, having approximately the same scale of spacing as that of the longitudinal grooves. A particularly interesting example offering additional evidence of a close link between longitudinal vortices and cross-hatching is the sequence of pictures in Fig. 14 of Ref. 5, showing transient ablation of a 50° Lexan cone. Cross-hatching appears first at a very small scale of spacing. Then the surface begins to ablate at a more rapid pace, wiping out the original cross-hatch pattern. A new pattern consisting of longitudinal grooves appears at a greater scale of spacing, and this is followed by the appearance of cross-hatching at the nose. The cross-hatching works its way rearward, overlaying the longitudinal grooves, and it is seen that the spacing of the cross-hatching is of the same order as that of the longitudinal grooves. Additional examples linking longitudinal vortices and cross-hatching can be found in Ref. 6.

Origin of Longitudinal Vortices

In Ref. 11, the mechanism triggering the appearance of longitudinal vortices is traced to the presence of small irregularities in the surface at or near the leading edge. We shall argue that these irregularities, in combination with a shear layer, are indeed capable of transforming otherwise two-dimensional boundary layers into boundary layers having significant concentrations of vorticity at spaced spanwise stations.

Lines of attachment and separation

In Fig. 4 is a planview of the surface showing, to an exaggerated scale, a pair of protuberances at the leading edge. Under certain conditions, a line of attachment[8] emanates from each of the stagnation points on the protuberances. An external streamline that attaches itself to the surface on a line of attachment then separates into a pair of limiting streamlines that diverge from either side of the line of attachment. Limiting streamlines diverging from the lines of attachment will tend to run into each other in the interval between the lines. Since they cannot cross, they will tend to converge along a line, a line of separation. Limiting streamlines from either side of the line of separation will combine and leave the surface on a surface of separation as single "separation streamlines."[8] We see that a pair of protuberances at the leading edge may cause the appearance between them of a line of separation where limiting streamlines can leave the surface.

Surface vortex lines

Now consider the surface vortex lines, a few of which are shown as dotted lines on Fig. 4. In steady flow, the surface vortex lines form an orthogonal net with the limiting streamlines. The protuberances cause the surface vortex lines to become distorted in their vicinity, a given surface vortex line developing a downstream-pointing loop aft of a protuberance and an upstream-pointing loop in the region between protuberances. We argue that, under certain conditions, the distortion of surface vortex lines near the leading edge will cause succeeding surface vortex lines to deform progressively; proceeding rearward, the surface vortex lines tend to develop a spanwise waveform. We shall see that if surface vortex lines do, in fact, become wavy, a consequence of this is the formation of additional lines of attachment and separation.

Assume that the flow is steady with respect to coordinates fixed in the body. Let us now move with a set of fluid particles which, at a time t_0, are at the position of the surface vortex line labeled (a) in Fig. 4. In moving coordinates, this surface vortex line is, in effect, attached to the particles and will move with them, undergoing successive deformations in time. The deformations will be just those of successive stationary surface vortex lines, proceeding rearward, as viewed in the body-fixed coordinate system. Consider the particle at the peak of one of the downstream-pointing loops in the surface vortex line. As shown on Fig. 5, the sense of the flow induced by segments of the vortex line on either side of the peak is such as to elevate the loop containing the particle. The elevated part of the vortex line will thereby tend to accelerate in the direction of flow, since the vortex line must move with the local stream velocity and the local stream velocity increases with increasing distance from the wall. (The opposite tends to occur near the peak of an upstream-pointing loop; here, the sense of the induced flow is such as to depress the vortex line, whereupon the depressed part of the line tends to move slower than the undeformed part.) Thus, the downstream-pointing loop tends to rise and to stretch and, as a consequence, the particle at the peak of the loop tends to follow a curved path, where the sense of the curvature is concave relative to the flow above. In the vertical plane containing this streamline, the situation is similar to that which obtains in the flow between rotating cylinders (cf. Ref. 14). The flow between rotating cylinders becomes centrifugally unstable when a critical value of the Taylor number is exceeded. In that case, if particles are displaced outward they continue to move outward, so that the streamlines followed by the particles run tangentially into the

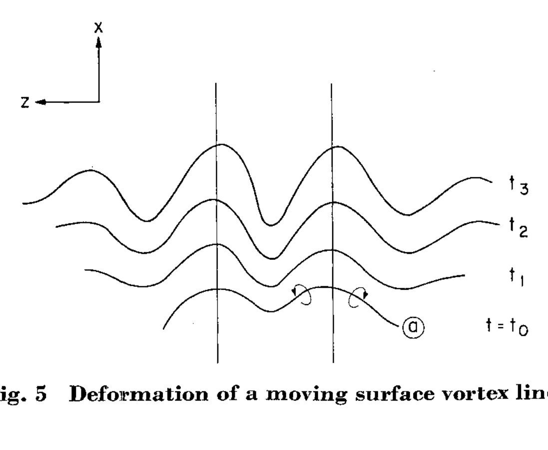

Fig. 5 Deformation of a moving surface vortex line.

limiting streamline next to the outer cylinder and thereby form a line of attachment. As shown in Ref. 14 by a physical argument, the Taylor number represents a ratio of centrifugal and viscous forces on a particle. Using a similar argument in our case, applied locally to the flow between the curved limiting streamline and an adjacent streamline above it, we arrive at a local Taylor number having the form

$$T_{\text{local}} = (hU_h/\nu)^2(h/R) \tag{8}$$

Here, h is a measure of a sublayer thickness over which U increases linearly with y, U_h is the local stream velocity at a distance h from the wall, ν is the kinematic viscosity near the wall, and R is the local radius of curvature of the limiting streamline. The magnitude of R will depend, among other things, on the size of the protuberance at the leading edge.† By analogy with the case of flow between rotating cylinders, we argue that if a critical value of the local Taylor number is exceeded, then external streamlines in the vertical plane containing the curved limiting streamline will turn downward and attach themselves tangentially to the limiting streamline, thereby forming a line of attachment.

Stretching of the surface vortex line (a) thus creates conditions leading to the formation of a line of attachment. Conversely, the formation of a line of attachment means that, in the vicinity of the line, the surface vortex line does in fact stretch progressively as shown in Fig. 5. The sense of the flow induced by the stretched part of the line on the undeformed part will be such as to cause the appearance of a new loop outboard of the original one. The new loop, in turn, will tend to stretch and then induce a further deformation of the line outboard of itself. Thus, as the surface vortex line proceeds downstream, the initial distortion will spread progressively outboard in the form of a wave.

Formation of longitudinal vortices

Now let us return to coordinates fixed in the body. As noted previously, the progressive deformation of a given surface vortex line moving downstream is identical to the deformation of successive fixed surface vortex lines proceeding rearward. These lines, the same as those shown in Fig. 5, are redrawn in Fig. 6. The limiting streamlines may now be drawn orthogonal to the surface vortex lines. We see that the deformation of the surface vortex lines into a waveform causes the appearance of a line of attachment at every locus of downstream-pointing loops and a line of separation at every locus of upstream-pointing loops in the surface vortex lines. Thus, the flow develops a cellular structure, wherein external streamlines attach themselves to the surface and limiting streamlines leave the surface on alternate vertical planes. This, in itself, does not invariably lead to the formation of longitudinal vortices, since, in principle, the streamlines leaving the surface can rise indefinitely. If, however, the streamlines of the external flow are approaching the body, or if they have concave curvature, then the streamlines leaving the surface will not be able to rise indefinitely, but rather will be turned back by the oncoming external flow. As shown in Fig. 6, a surface of separation will end at a stagnation line,

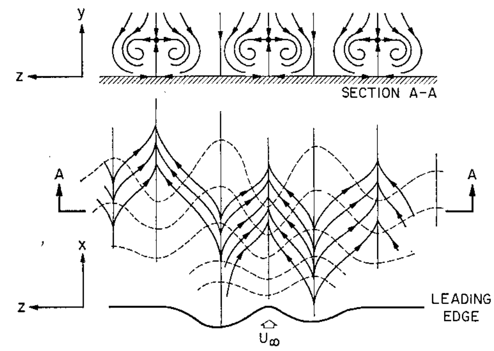

Fig. 6 Formation of longitudinal vortices.

and the flow adjacent to the surface of separation will turn back to form a pair of counterrotating longitudinal vortices.‡

As formulated, the mechanism causing the occurrence of longitudinal vortices does not depend on the state of the boundary layer, i.e., whether it is laminar, transitional, or turbulent. The lines of attachment and separation, which lay the groundwork for the formation of vortices, lie deep within the viscous sublayer and may be presumed to exist no matter what the state of the boundary layer elsewhere.

We have argued that a surface irregularity in the form of a pair of protuberances at the leading edge can cause the formation downstream of a stationary array of counterrotating longitudinal vortices. It is possible to extend the argument to apply in the case of a roughness element on the surface itself. Finally, a random collection of roughness elements near the leading edge likewise may be expected to produce a similar result. No matter what the form of distortion of the surface vortex lines near the leading edge, further downstream, the surface vortex lines will again tend to adopt a waveform. The remainder of the argument then follows as outlined previously.

Sources of Streamline Curvature

The capacity of surface irregularities to cause the formation of regular arrays of counterrotating longitudinal vortices is seen to be enhanced by the presence of concave curvature in the streamlines at the outer edge of the boundary layer. Here we enumerate some of the sources of streamline curvature available in the experiments where cross-hatching has been observed.

It is convenient first to refer again to Ginoux's experiments involving boundary-layer flows over flat plates with backward-facing steps.[11] Here, the evidence is clear that longitudinal vortices are in fact formed. In Ginoux's experiments, the source of streamline curvature is the backward-facing step. The boundary-layer streamlines, in separating from the corner and reattaching on the plate, clearly sustain a large concave curvature. The same source is available and even accentuated in all of the wind-tunnel models of Refs. 3–7 that had metal tips (Fig. 7a). When the model begins to ablate, a step appears at the juncture of the tip and the ablating surface. Just as in Ginoux's experiments, this alone should be sufficient to cause the necessary curvature of the streamlines. Additional curvature is available, however, in that the surface behind the tip, ablating at a higher rate than else-

† Another factor is curvature of the wall itself. Görtler's classical analysis of boundary-layer instability over concave walls applies to this case.[15] Görtler's theory can be used to estimate the amount of curvature of the limiting streamline required to exceed a critical value of the Taylor number. For conditions approximating those of the experiments of Larson and Mateer,[4,5] the estimated required radius of curvature is of the order of 100m. We have derived a correction accounting approximately for the effect of compressibility which, under the same conditions, reduces the required radius of curvature to the order of 20m. The requirement becomes more stringent but the conclusion remains the same, namely, that practically imperceptible curvature is all that is required to exceed the critical Taylor number.

‡ The flow adjacent to surfaces of separation may turn back to form longitudinal vortices even in the absence of a restraining external flow (e.g., the flat plate with zero pressure gradient). The Reynolds stress may provide the necessary restraining influence in such cases. The appearance of longitudinal vortices in the boundary layer over flat plates, often observed experimentally,[16,17] and known to play a critical role in boundary-layer transition[18,19] may be explainable on this basis. In any case, it is believed that the key to the common appearance of longitudinal vortices is the readiness of surface vortex lines to adopt a waveform.

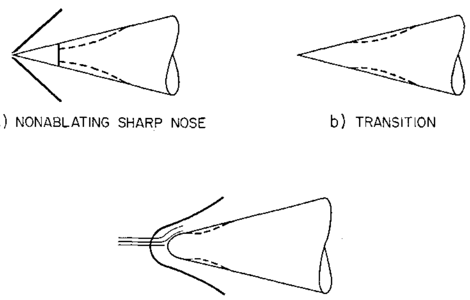

Fig. 7 Sources of streamline curvature.

where, becomes concave. The surface behind the tip ablates at a higher rate because heating increases where the flow is compressed after reattaching.

The same phenomenon of increased heating in a local region causing or accentuating a surface concavity may occur through a number of mechanisms. Recompression of the flow is one. Another is boundary-layer transition on an ablating surface (Fig. 7b).

For conical bodies having blunt noses, or whose noses become blunt as a result of ablation, several sources of concave streamline curvature exist. One is the already mentioned local region of increased heating. As shown on Fig. 7c (cf. Ref. 20), the flow undergoes a compression in negotiating the blunt nose, so that, for an ablating body, a local region of increased heating is available to cause a surface concavity. For both conical bodies and wedges, the blunt nose itself may provide the necessary curvature of the streamlines in the vicinity of the stagnation point (cf. Fig. 7c). This source of curvature has already been discussed by Görtler[21] and early experimental evidence suggesting that longitudinal vortices are in fact formed is given in Ref. 22. More recently, experiments with sublimating cylinders[23] have shown clear evidence of the presence of longitudinal vortices on the forward part of the cylinders.

Concluding Remarks

The hypothesis advanced here may be summarized as follows: cross-hatching is the result of spatially periodic variations in surface pressure in both the spanwise and longitudinal directions. The source of the pressure variations is the presence in the boundary layer of an array of regularly spaced counterrotating longitudinal vortices. The vortices originate from surface irregularities near the leading edge; the probability of their appearance is enhanced by the existence of small amounts of concave curvature of the boundary-layer streamlines. Surface ablation is not a necessary condition for the presence of the pressure variations that lead to cross-hatching, but may serve as the mechanism causing the streamline curvature and as a means of reinforcing and spreading the crosshatch pattern once it appears.

The hypothesis can be tested by recording temperatures or pressures on the surface of a nonablating body, the shape of which introduces a local concavity in the boundary-layer streamlines. Aft of the concavity, the boundary layer must be turbulent and the boundary-layer-edge Mach number greater than unity. The detection of longitudinal vortices in the boundary layer aft of the concavity and surface temperature or pressure maxima at spaced points connected by Mach lines, the spacing having the same scale as that of the vortices, would substantiate the hypothesis.

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